

Tree-Level Nondecoupling and the Supersymmetric Higgs Sector

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Abstract

Because of the existence of cubic scalar couplings, there are in general nondecoupling effects at tree level in the scalar sector of any theory with two or more very different mass scales. We show this explicitly in the minimal nonsupersymmetric SU(5) model of grand unification. We show also how tree-level decoupling is guaranteed if supersymmetry is imposed. On the other hand, if the gauge symmetry is larger than that of the standard model at the mass scale of supersymmetry breaking, the two-Higgs-doublet structure at the presumably lower electroweak energy scale will be different from that of the minimal supersymmetric standard model, as shown already previously in a number of specific examples. We add here one example involving four Higgs doublets.

1 Introduction

In the study of fundamental interactions, it is important to recognize the relevant energy scale or scales of the specific processes being discussed. In quantum field theory, it means knowing whether an interaction at a particular energy scale has an inseparable component from physics at a much higher scale. If not, then there is decoupling, and the theoretical interpretation of any experimental result becomes tractable in the context of that theory. On the other hand, if there is nondecoupling, then an irreducible degree of uncertainty must always remain.

In the standard electroweak gauge model, although there is really only one energy scale, *i.e.* $v = (2\sqrt{2}G_F)^{-1/2} \simeq 170$ GeV, nondecoupling is known to occur in the limit of large fermion masses. We review this in Sec. 2 and show that in addition to the commonly touted loop effects, nondecoupling is already present at tree level. We then discuss in Sec. 3 the case of two very different energy scales and show how nondecoupling occurs at tree level in the Higgs sector with two explicit examples: one of $SU(5)$ breaking down to $SU(3) \times SU(2) \times U(1)$, and the other of two scalar doublets in the standard model. In Sec. 4 we show how exact supersymmetry guarantees the decoupling of the two scales. In Sec. 5, we show how softly broken supersymmetry allows nondecoupling and apply it to the case where the gauge symmetry is larger than that of the standard model at the mass scale of supersymmetry breaking. As shown in several previous specific examples[1, 2, 3], the two-Higgs-doublet structure at the presumably lower electroweak energy scale will be different from that of the minimal supersymmetric standard model (MSSM). We add here one example involving four Higgs doublets. Finally in Sec. 6, there are some concluding remarks.

2 Nondecoupling in the Standard Model

The decoupling[4] of particles heavier than a certain mass scale from physics at a much lower energy is important for the proper interpretation of experimental observables in terms of a particular theory. In the standard electroweak gauge model, there is technically only one scale, *i.e.* the vacuum expectation value of the neutral component of the Higgs scalar doublet $v = (2\sqrt{2}G_F)^{-1/2} \simeq 174$ GeV. Whereas all masses in this model are proportional to v , it is still meaningful to consider the limit that one of these masses is much larger than all the others, and ask if the former's contribution to physically measurable quantities vanishes or not. It has been known for some time that nondecoupling of heavy particles does occur[5] in models with spontaneous symmetry breaking, and since the heaviness of the t quark is now established, *i.e.* $m_t = 180 \pm 12$ GeV[6], its contributions to many observables are confirmed to be nonvanishing and nonnegligible. The lesson we learn here is that without knowing the value of m_t , there would be large uncertainties in the interpretation of data in terms of the standard model.

Examples of nondecoupling in the standard model abound, but they have been invariably given as loop effects. Consider the process $H \rightarrow \gamma\gamma$, where H is the standard-model Higgs boson. Since the Yukawa coupling of H to $t\bar{t}$ is proportional to m_t/v , the t contribution to this amplitude is of the form

$$\frac{m_t}{v} \times \frac{e^2}{16\pi^2} \frac{1}{m_t}, \quad (1)$$

which goes to a nonzero constant as $m_t \rightarrow \infty$. In other words, the suppression of the loop due to a large m_t is exactly compensated by the increased coupling. If t is replaced with the W boson, then we have instead

$$g^2 v \times \frac{e^2}{16\pi^2} \frac{1}{g^2 v^2}, \quad (2)$$

which is again finite as $M_W = gv/\sqrt{2}$ goes to infinity, because we must hold v finite to

get a finite m_H . In the above, there is always the implicit but crucial assumption that $m_H \ll m_t$ and $m_H \ll M_W$, because we are concerned with the effect of heavy particles on experimental observables far below the energy required to produce these heavy particles.

Another example is the one-loop radiative correction to the W and Z self-energies. The oblique parameter T [7] is proportional to the difference between $\Pi_{11}(0) = \Pi_{22}(0)$ and $\Pi_{33}(0)$, which has a contribution from the t and b quarks of the form

$$\frac{g^2}{16\pi^2} \left[m_t^2 + m_b^2 - \frac{2m_t^2 m_b^2 \ln(m_t^2/m_b^2)}{m_t^2 - m_b^2} \right], \quad (3)$$

which is always nonnegative and is zero only if $m_t = m_b$. It also increases without bound as m_t (or m_b) increases. The implicit assumption here is that the W and Z masses are fixed.

Actually, nondecoupling in the standard model already occurs at tree level. Consider the interaction of four light particles. Divide them into two pairs. If a heavy particle (of mass M) couples to each pair, then an effective coupling of the form $g_1 g_2 / M^2$ appears. In the standard model, in the case of four light fermions, $g_1 = g_2 = g/2\sqrt{2}$ and $M^2 = g^2 v^2/2$, hence the effective coupling is $1/4v^2$ which does not vanish as $M \rightarrow \infty$. This is just like the previous example regarding T , except now the Yukawa coupling f in $m_f = f v$ is assumed small compared to the gauge coupling g instead of the other way around.

3 Nondecoupling in the Case of Two Scales

Consider now a gauge group larger than that of the standard model. Let the former break down to the latter at the scale v_H which is much larger than the electroweak scale v . Let H be a heavy scalar boson (with mass proportional to v_H) which is a singlet under the standard $SU(3) \times SU(2) \times U(1)$ gauge symmetry. Let Φ be the usual standard Higgs doublet and assume that the cubic interaction $\Phi^\dagger \Phi H$ exists. Now if this coupling strength is proportional

to v_H also, then the effective $(\Phi^\dagger\Phi)^2$ coupling is of the form

$$v_H \frac{1}{v_H^2} v_H, \quad (4)$$

which does not vanish as $v_H \rightarrow \infty$. Here the nondecoupling of the heavy particle H has occurred at tree level in spite of the large ratio v_H/v , in contrast to the last example in the previous section where the ratio g/f is large but there is only one scale.

In a nonsupersymmetric quantum field theory with two (or more) scales, the above nondecoupling phenomenon in the scalar sector is a general occurrence. In the next section, we will show how an exactly supersymmetric theory enforces the decoupling of the two scales. Here we provide first a very useful example of nonsupersymmetric $SU(5)$ breaking into $SU(3) \times SU(2) \times U(1)$ in the presence of one adjoint **24** and one fundamental **5** of Higgs scalar representations. It is often implicitly assumed here that electroweak symmetry breaking is determined by the quartic self-interaction of the doublet scalar field Φ in the **5** which appears in the $SU(5)$ Lagrangian. However, it will be shown in the following that because of nondecoupling contributions from heavy particles contained in the **24**, such is not the case. One important consequence of this result is that if one uses the renormalization group equations to run the former coupling from the $SU(5)$ scale to the electroweak scale, it will not be the experimentally observed coupling.

Let the adjoint **24** scalar representation be denoted by a 5×5 matrix:

$$H = \begin{bmatrix} H_{\alpha\beta} - (2/15)^{1/2} H_0 \delta_{\alpha\beta} & H_{\alpha j} \\ H_{i\beta} & H_{ij} + (3/10)^{1/2} H_0 \delta_{ij} \end{bmatrix}, \quad (5)$$

where $\alpha, \beta = 1, 2, 3$; $i, j = 4, 5$; and

$$H_{ij} = 2^{-1/2} \vec{\tau} \cdot \vec{H}_3, \quad H_{\alpha\beta} = 2^{-1/2} \vec{\lambda} \cdot \vec{H}_8. \quad (6)$$

In the above, $\vec{\tau}$ denotes the 3 $SU(2)$ 2×2 representation matrices and $\vec{\lambda}$ the 8 $SU(3)$ 3×3

ones. The vacuum expectation value of H is assumed to be given by

$$\langle H \rangle = \frac{\langle H_0 \rangle}{\sqrt{30}} \begin{bmatrix} -2 & & & & \\ & -2 & & & \\ & & -2 & & \\ & & & 3 & \\ & & & & 3 \end{bmatrix} + \frac{\langle H_3^0 \rangle}{\sqrt{2}} \begin{bmatrix} 0 & & & & \\ & 0 & & & \\ & & 0 & & \\ & & & 1 & \\ & & & & -1 \end{bmatrix}. \quad (7)$$

Note that the usual discussion of $SU(5)$ symmetry breaking routinely neglects the triplet scalar field \vec{H}_3 as well as its vacuum expectation value. However, it will be shown in the following that they are essential in correctly understanding the symmetry breaking. The fundamental $\mathbf{5}$ is denoted by $\Phi = [\Phi_\alpha, \Phi_i]$, with $\Phi_i = (\phi^+, \phi^0)$; hence $\langle \Phi \rangle = \langle \phi^0 \rangle [0, 0, 0, 0, 1]$.

The most general Higgs potential consisting of H and Φ which is also invariant under the discrete symmetry $H \rightarrow -H$ is given by

$$\begin{aligned} V = & \frac{1}{2} m_1^2 \text{Tr} H^2 + \frac{1}{4} \lambda_1 (\text{Tr} H^2)^2 + \frac{1}{4} \lambda_2 \text{Tr} H^4 \\ & + m_2^2 \Phi^\dagger \Phi + \frac{1}{2} \lambda_3 (\Phi^\dagger \Phi)^2 + \lambda_4 (\text{Tr} H^2) (\Phi^\dagger \Phi) + \lambda_5 (\Phi^\dagger H^2 \Phi). \end{aligned} \quad (8)$$

Let

$$\langle H_0 \rangle = v_1, \quad \langle \phi^0 \rangle = v_2 / \sqrt{2}, \quad \langle H_3^0 \rangle = v_3, \quad (9)$$

then the minimum of V satisfies

$$v_1 [m_1^2 + (\lambda_1 + \frac{7}{30} \lambda_2) v_1^2 + (\lambda_4 + \frac{3}{10} \lambda_5) v_2^2 + (\lambda_1 + \frac{9}{10} \lambda_2) v_3^2] - \frac{1}{2} \sqrt{\frac{3}{5}} \lambda_5 v_2^2 v_3 = 0, \quad (10)$$

$$v_2 [m_2^2 + (\lambda_4 + \frac{3}{10} \lambda_5) v_1^2 + \frac{1}{2} \lambda_3 v_2^2 - \sqrt{\frac{3}{5}} \lambda_5 v_1 v_3 + \frac{1}{2} \lambda_5 v_3^2] = 0, \quad (11)$$

$$v_3 [m_1^2 + (\lambda_1 + \frac{9}{10} \lambda_2) v_1^2 + (\lambda_4 + \frac{1}{2} \lambda_5) v_2^2 + (\lambda_1 + \frac{1}{2} \lambda_2) v_3^2] - \frac{1}{2} \sqrt{\frac{3}{5}} \lambda_5 v_2^2 v_1 = 0. \quad (12)$$

Note that $v_1 \neq 0, v_2 = v_3 = 0$ is a solution; but if $v_2 \neq 0$ as well, then $v_3 \neq 0$ necessarily. The often quoted naive solution without v_3 is not strictly correct, but since its magnitude is of order v_2^2/v_1 , it is negligible for all known purposes.

Solving the above 3 equations, using the valid approximation that $v_1 \gg v_2 \gg v_3$, we obtain

$$v_1^2 \simeq \frac{-m_1^2}{\lambda_1 + 7\lambda_2/30}, \quad (13)$$

and

$$v_2^2 \simeq \frac{-m_2^2 + (\lambda_4 + 3\lambda_5/10)m_1^2/(\lambda_1 + 7\lambda_2/30)}{\lambda_3/2 - 9\lambda_5^2/20\lambda_2 - (\lambda_4 + 3\lambda_5/10)^2/(\lambda_1 + 7\lambda_2/30)}. \quad (14)$$

Let us go back to V and consider the $\Phi_i^\dagger \Phi_i$ term. This is correctly given in the usual treatment as

$$\mu^2 = m_2^2 + \lambda_4 v_1^2 + \frac{3}{10} \lambda_5 v_1^2, \quad (15)$$

but then it is often claimed that

$$v_2^2 = \frac{-\mu^2}{\lambda_3/2} = \frac{-m_2^2 + (\lambda_4 + 3\lambda_5/10)m_1^2/(\lambda_1 + 7\lambda_2/30)}{\lambda_3/2}, \quad (16)$$

which is of course wrong. The two missing terms in the denominator are exactly those given by the nondecoupling contributions of H_0 and \vec{H}_3 . It is easy to verify that H_0 has mass-squared $= 2(\lambda_1 + 7\lambda_2/30)v_1^2$ and its coupling to $\Phi_i^\dagger \Phi_i$ is $2(\lambda_4 + 3\lambda_5/10)v_1$; and \vec{H}_3 has mass-squared $= (2\lambda_2/3)v_1^2$ and its coupling to $\Phi_i^\dagger \vec{\tau}_{ij} \Phi_j$ is $\sqrt{3/5}\lambda_5 v_1$. Hence the correct quartic self-coupling of the Higgs doublet Φ at low energy is

$$\lambda_3 - \frac{9}{10} \frac{\lambda_5^2}{\lambda_2} - \frac{2(\lambda_4 + 3\lambda_5/10)^2}{\lambda_1 + 7\lambda_2/30}. \quad (17)$$

In other words, there are inseparable contributions from heavy particles at the large scale v_1 to the low-energy interactions of light particles at the small scale v_2 . The only way that these contributions can be discovered is to increase the experimental energy scale up to v_1 .

Another demonstration of this kind of tree-level nondecoupling is available in the well-known extension of the standard model to include two Higgs doublets. Let the Higgs potential be given by

$$\begin{aligned} V = & \mu_1^2 \Phi_1^\dagger \Phi_1 + \mu_2^2 \Phi_2^\dagger \Phi_2 + \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 \\ & + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) + \frac{1}{2} \lambda_5 [(\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2]. \end{aligned} \quad (18)$$

Let $\langle \phi_{1,2}^0 \rangle = v_{1,2}$, then

$$v_1[\mu_1^2 + \lambda_1 v_1^2 + (\lambda_3 + \lambda_4 + \lambda_5)v_2^2] = 0, \quad (19)$$

$$v_2[\mu_2^2 + \lambda_2 v_2^2 + (\lambda_3 + \lambda_4 + \lambda_5)v_1^2] = 0. \quad (20)$$

Suppose $v_2 \ll v_1$, then $v_1^2 \simeq -\mu_1^2/\lambda_1$ and

$$v_2^2 \simeq \frac{-\mu_2^2 + (\lambda_3 + \lambda_4 + \lambda_5)\mu_1^2/\lambda_1}{\lambda_2 - (\lambda_3 + \lambda_4 + \lambda_5)^2/\lambda_1}. \quad (21)$$

Again, the heavy ϕ_1^0 contribution is nondecoupling. An important note is that the two scales v_1 and v_2 can be separated in principle here because V has a discrete symmetry $\Phi_1 \rightarrow \Phi_1$, $\Phi_2 \rightarrow -\Phi_2$, which is unbroken in the case $v_1 \neq 0$, $v_2 = 0$.

4 Guarantee of Decoupling in Supersymmetry

In a supersymmetric quantum field theory, the Higgs potential V is necessarily nonnegative. However, there may be several minima, each having $V = 0$ but corresponding to different sets of vacuum expectation values. Hence the gauge symmetry may be broken while the supersymmetry is preserved. For example in the case supersymmetric $SU(5)$, it has been shown[8] that with an adjoint **24**, the supersymmetric-preserving vacuum may be symmetric under $SU(5)$ (*i.e.* no breaking), $SU(4) \times U(1)$, or $SU(3) \times SU(2) \times U(1)$ (*i.e.* the standard model). [In the notation of the previous section, these solutions correspond to $v_1 = v_3 = 0$; $v_1 = 0$, $v_3 \neq 0$; and $v_1 \neq 0$, $v_3 = 0$ respectively.] To understand how exact supersymmetry guarantees the decoupling of heavy particles from low-energy physics, we need only consider the structure of V given by the superpotential W , as follows.

To be specific, consider the two usual doublet superfields of the supersymmetric standard model:

$$\tilde{\Phi}_1 \equiv i\tau_2 \Phi_1^* = \begin{pmatrix} \bar{\phi}_1^0 \\ -\phi_1^- \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}. \quad (22)$$

Introduce the heavy superfields S and $\vec{\Sigma}$ (singlet and triplet respectively under the standard electroweak gauge group). Then the relevant superpotential involving the above is given by

$$W = \frac{1}{2}m_S S S + \frac{1}{2}m_\Sigma \vec{\Sigma} \cdot \vec{\Sigma} + \mu \Phi_1^\dagger \Phi_2 + f_S S \Phi_1^\dagger \Phi_2 + \frac{1}{2}f_\Sigma \Phi_1^\dagger (\vec{\Sigma} \cdot \vec{\tau}) \Phi_2, \quad (23)$$

where $\mu \ll m_S, m_\Sigma$. If supersymmetry is exact, then the part of the Higgs potential which comes from W is given by

$$\begin{aligned} V_F = & |m_S S + f_S \Phi_1^\dagger \Phi_2|^2 + |m_\Sigma \vec{\Sigma} + \frac{1}{2}f_\Sigma \Phi_1^\dagger \vec{\tau} \Phi_2|^2 \\ & + |f_S S \Phi_2 + \frac{1}{2}f_\Sigma \vec{\Sigma} \cdot \vec{\tau} \Phi_2 + \mu \Phi_2|^2 + |f_S \Phi_1^\dagger S + \frac{1}{2}f_\Sigma \Phi_1^\dagger \vec{\Sigma} \cdot \vec{\tau} + \mu \Phi_1^\dagger|^2, \end{aligned} \quad (24)$$

which is clearly nonnegative. Now the cubic $\Phi_{1,2}^\dagger \Phi_{1,2} S$ coupling strength is $f_S \mu$, hence the S contribution to $(\Phi_{1,2}^\dagger \Phi_{1,2})^2$ is of the form

$$f_S \mu \frac{1}{m_S^2} f_S \mu, \quad (25)$$

which goes to zero as $m_S/\mu \rightarrow \infty$. On the other hand, the cubic $\Phi_1^\dagger \Phi_2 S$ coupling strength is $f_S m_S$ which is large, but V_F also contains an explicit $|\Phi_1^\dagger \Phi_2|^2$ term, hence the S contribution here is given by

$$f_S^2 - f_S m_S \frac{1}{m_S^2} f_S m_S, \quad (26)$$

which is zero. Similarly, the $\vec{\Sigma}$ contributions also decouple. In fact, the only term which survives is $\mu^2(\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2)$. There is also a part of V which comes from the gauge sector but it has no cubic interactions and thus no additional tree-level contributions from the heavy scalar fields. In the notation of Eq. (18), the quartic scalar couplings are given by

$$\lambda_1 = \lambda_2 = \frac{1}{4}(g_1^2 + g_2^2), \quad \lambda_3 = -\frac{1}{4}g_1^2 + \frac{1}{4}g_2^2, \quad \lambda_4 = -\frac{1}{2}g_2^2, \quad \lambda_5 = 0. \quad (27)$$

This means that as long as supersymmetry is maintained exactly above the electroweak symmetry breaking scale, the two-Higgs-doublet structure is uniquely given by the minimal supersymmetric standard model (MSSM). However, the scale of soft supersymmetry breaking

M_{SUSY} may be somewhat higher, say a few TeV instead of 100 GeV, so there is room enough for a larger gauge symmetry to be in effect above M_{SUSY} . It must of course also break down to the standard $SU(3) \times SU(2) \times U(1)$ below M_{SUSY} . In this case, the nondecoupling of heavy particles at the M_{SUSY} scale will change the quartic scalar couplings listed above, as already shown in several previous explicit examples[1, 2, 3].

5 Nondecoupling with Softly Broken Supersymmetry

Consider the following scenario of symmetry breaking:

energy	gauge group	supersymmetry
10^{16} GeV	$G \rightarrow G'$	unbroken
10^x GeV	$G' \rightarrow G''$	unbroken
10^3 GeV	$G'' \rightarrow G_{SM}$	broken
10^2 GeV	$G_{SM} \rightarrow SU(3) \times U(1)$	broken

In the above, G_{SM} is the standard-model gauge group and 10^x GeV is a possible but unknown intermediate scale. The usual assumption is that $G'' = G_{SM}$ in which case supersymmetry would protect the MSSM up to 10^x GeV. Often it is also assumed that $G' = G''$, in which case $x = 16$. However, if G'' is larger than G_{SM} , then the physics at 10^2 GeV will have nondecoupling contributions from G'' .

In addition to previous examples[1, 2, 3] of two Higgs doublets at the electroweak scale which are not those of the MSSM, consider here a model which ends up with four Higgs doublets. It is the supersymmetric version of a gauge model of generation nonuniversality proposed many years ago[9]. We extend the electroweak gauge group to $SU(2)_{12} \times SU(2)_3 \times U(1)$ with couplings g_{12} , g_3 , and g_0 , such that left-handed quark and lepton doublets of the first two generations couple to $SU(2)_{12}$ but those of the third couple to $SU(2)_3$. To make this into a supersymmetric theory, in analogy to the doubling of Higgs scalars in the standard

model, we need the following scalar multiplets:

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix} \sim (2, 1, \frac{1}{2}), \quad \Phi_2 = \begin{pmatrix} \bar{\phi}_2^0 \\ -\phi_2^- \end{pmatrix} \sim (2, 1, -\frac{1}{2}), \quad (28)$$

$$\Phi_3 = \begin{pmatrix} \phi_3^+ \\ \phi_3^0 \end{pmatrix} \sim (1, 2, \frac{1}{2}), \quad \Phi_4 = \begin{pmatrix} \bar{\phi}_4^0 \\ -\phi_4^- \end{pmatrix} \sim (1, 2, -\frac{1}{2}), \quad (29)$$

$$\eta = \begin{pmatrix} \bar{\eta}_2^0 & \eta_1^+ \\ -\eta_2^- & \eta_1^0 \end{pmatrix} \sim (2, 2, 0), \quad S \sim (1, 1, 0). \quad (30)$$

The spontaneous breaking of $SU(2)_{12} \times SU(2)_3$ to $SU(2)_{SM}$ is achieved by having $\langle \eta_1^0 \rangle = \langle \bar{\eta}_2^0 \rangle = u$. The singlet S is added in the above because a careful examination of the Higgs potential shows that without it, there would be no such solution. From the gauge interactions alone, the Higgs potential is given by

$$\begin{aligned} V_D &= \frac{1}{8}g_0^2(\Phi_1^\dagger\Phi_1 - \Phi_2^\dagger\Phi_2 + \Phi_3^\dagger\Phi_3 - \Phi_4^\dagger\Phi_4)^2 \\ &+ \frac{1}{8}g_{12}^2 \sum_a (\Phi_1^\dagger\tau^a\Phi_1 + \Phi_2^\dagger\tau^a\Phi_2 + Tr\eta^\dagger\tau^a\eta)^2 \\ &+ \frac{1}{8}g_3^2 \sum_a (\Phi_3^\dagger\tau^a\Phi_3 + \Phi_4^\dagger\tau^a\Phi_4 - Tr\eta\tau^a\eta^\dagger)^2. \end{aligned} \quad (31)$$

From the superpotential

$$\begin{aligned} W &= \mu_1\tilde{\Phi}_1^\dagger\Phi_2 + \mu_2\tilde{\Phi}_3^\dagger\Phi_4 + \mu_3Tr\tilde{\eta}^\dagger\eta + \frac{1}{2}\mu_4S^2 + f_1\tilde{\Phi}_1^\dagger\eta\Phi_4 + f_2\tilde{\Phi}_2^\dagger\eta\Phi_3 \\ &+ \lambda_1S\tilde{\Phi}_1^\dagger\Phi_2 + \lambda_2S\tilde{\Phi}_3^\dagger\Phi_4 + \lambda_3STr\tilde{\eta}^\dagger\eta + \frac{1}{3}\lambda_4S^3, \end{aligned} \quad (32)$$

we obtain

$$\begin{aligned} V_F &= |\lambda_1\tilde{\Phi}_1^\dagger\Phi_2 + \lambda_2\tilde{\Phi}_3^\dagger\Phi_4 + \lambda_3Tr\tilde{\eta}^\dagger\eta + \lambda_4S^2 + \mu_4S|^2 \\ &+ \sum_{i,j} |2\mu_3\tilde{\eta}_{ji}^\dagger + 2\lambda_3S\tilde{\eta}_{ji}^\dagger + f_1\tilde{\Phi}_{1i}^\dagger\Phi_{4j} + f_2\tilde{\Phi}_{2i}^\dagger\Phi_{3j}|^2 \\ &+ |\mu_1\Phi_2 + \lambda_1S\Phi_2 + f_1\eta\Phi_4|^2 + |-\mu_1\Phi_1 - \lambda_1S\Phi_1 + f_2\eta\Phi_3|^2 \\ &+ |-\mu_2\tilde{\Phi}_4^\dagger - \lambda_2S\tilde{\Phi}_4^\dagger + f_2\tilde{\Phi}_2^\dagger\eta|^2 + |\mu_2\tilde{\Phi}_3^\dagger + \lambda_2S\tilde{\Phi}_3^\dagger + f_1\tilde{\Phi}_1^\dagger\eta|^2, \end{aligned} \quad (33)$$

where $\tilde{\eta} \equiv \tau_2\eta^*\tau_2$, and $\tilde{\Phi}_1^\dagger\Phi_2 = -\tilde{\Phi}_2^\dagger\Phi_1$ has been used.

As η and S acquire vacuum expectation values $u_1 = u_2 = u$ and s respectively, this model reduces to the standard model as far as the gauge interactions are concerned. All left-handed quarks and leptons are now doublets under $SU(2)_{SM}$ with coupling strength g_{123} given by[9]

$$g_{123}^{-2} = g_{12}^{-2} + g_3^{-2}. \quad (34)$$

In the Higgs sector, the 8 scalar fields contained in the bidoublet η are now organized into a massless triplet (*i.e.* the would-be Goldstone bosons of this symmetry breaking), a massive triplet $[Re(\eta_1^0 - \eta_2^0), (\eta_1^\pm - \eta_2^\pm)/\sqrt{2}]$, and two singlets, *i.e.* $Re(\eta_1^0 + \eta_2^0)$ and $Im(\eta_1^0 - \eta_2^0)$, the first of which also mixes with ReS . Assume for simplicity

$$V_{soft} = \mu^2 Tr \eta^\dagger \eta + m^2 |S|^2, \quad (35)$$

then the triplet $[Re(\eta_1^0 - \eta_2^0), (\eta_1^\pm - \eta_2^\pm)/\sqrt{2}]$ has mass-squared given by

$$M^2 = (g_{12}^2 + g_3^2)u^2 - 4\lambda_3(2\lambda_3 u^2 + \lambda_4 s^2 + \mu_4 s), \quad (36)$$

and couples to $\Phi_{1,2}^\dagger \vec{\tau} \Phi_{1,2}$ with strength $-(1/2)g_{12}^2 u$, and to $\Phi_{3,4}^\dagger \vec{\tau} \Phi_{3,4}$ with strength $(1/2)g_3^2 u$.

The effective $(\Phi_{1,2}^\dagger \Phi_{1,2})^2$ interaction is thus

$$\frac{1}{2}g_{12}^2 - \frac{2(g_{12}^2 u/2)^2}{M^2}, \quad (37)$$

which reduces to $g_{123}^2/2$ if we drop terms in M^2 having to do with the superpotential. The same result holds for the effective $(\Phi_{3,4}^\dagger \Phi_{3,4})^2$ interaction with the interchange of g_{12}^2 and g_3^2 . Other nondecoupling contributions also appear, but their cubic interactions do not involve the gauge couplings. The four-doublet structure of the reduced Higgs potential has thus many parameters and is not simply a function of the standard-model gauge couplings.

6 Concluding Remarks

We have demonstrated in this paper that tree-level nondecoupling occurs generally in the scalar sector of a spontaneously broken gauge theory. The origin is the presence of cubic

couplings of the form $\Phi_i^\dagger \Phi_j H$ where H is a heavy scalar field. If this coupling strength is of order m_H , then there is a nondecoupling contribution to the low-energy $|\Phi_i^\dagger \Phi_j|^2$ interaction. This does not occur in a supersymmetric theory, because this contribution is exactly canceled by an existing $|\Phi_i^\dagger \Phi_j|^2$ coupling, or the cubic coupling strength itself is much smaller than m_H .

We show in particular the nondecoupling of the quartic self-coupling of the standard-model Higgs doublet from superheavy scalar bosons in a nonsupersymmetric $SU(5)$ grand unified theory. This has the important implication that the usual renormalization-group analysis of the evolution of this coupling from the $SU(5)$ scale to the electroweak scale is not valid. On the other hand, in a supersymmetric field theory, there is no such problem. This is easily understood because supersymmetry relates quartic scalar couplings to gauge and Yukawa couplings. Since the latter do not have nondecoupling contributions, neither must the former.

Turning the argument around, we emphasize the possibility that if the scale of soft supersymmetry breaking is a few TeV and there exists a gauge symmetry larger than that of the standard model just above it, then nondecoupling of the TeV-scale physics from the electroweak Higgs sector may occur. This has been shown explicitly in several previous examples[1, 2, 3]. It has the important implication that if two Higgs doublets are found at the electroweak scale and they are not those of the minimal supersymmetric standard model (MSSM), it does not rule out the existence of supersymmetry. There is a large class of supersymmetric theories with an extended gauge symmetry at the TeV scale, which has two Higgs doublets at the electroweak scale different from those of the MSSM. We add one more example here in this paper involving four Higgs doublets. Finally we should remark that our results are of course based on perturbation theory, but there can also be nonperturbative nondecoupling effects through instantons in certain theories[10].

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